***SOLUTION Section* 3.6 – Alternating Series, Absolute and Conditional Convergence**

***Exercise***

Determine if the alternating series converges or diverges 

***Solution***









Therefore; the series  ***converges*** by *Alternating Convergence Test*.

***Exercise***

Determine if the alternating series converges or diverges 

***Solution***













Therefore; the series  ***converges*** by *Alternating Series Test*.

***Exercise***

Determine if the alternating series converges or diverges 

***Solution***















Therefore; the series  ***converges*** by *Alternating Series Test*.

***Exercise***

Determine if the alternating series converges or diverges 

***Solution***



The given series ***diverges*** by nth Term Test for Divergence.

***Exercise***

Determine if the alternating series converges or diverges 

***Solution***



Therefore; the given series ***diverges*** by n*th* *Term Test* for Divergence.

***Exercise***

Determine if the alternating series converges or diverges 

***Solution***











Therefore; the given series ***converges*** by *Alternating Series Test*.

***Exercise***

Determine if the alternating series converges or diverges 

***Solution***







Therefore; the given series ***converges*** by *Alternating Series Test*.

***Exercise***

Determine if the alternating series converges or diverges 

***Solution***



Therefore; the given series ***converges*** by n*th*-Term Test.

***Exercise***

Determine if the alternating series converges or diverges 

***Solution***







Therefore; the given series ***converges*** by Alternating Series Test. 

***Exercise***

Determine if the alternating series converges or diverges

***Solution***







Therefore; the given series ***converges*** by Alternating Series Test. 

***Exercise***

Determine if the alternating series converges or diverges 

***Solution***



Therefore; the given series ***diverges*** by n*th*-Term Test.

***Exercise***

Determine if the alternating series converges or diverges

***Solution***

 ***or*** 





Therefore; the given series ***converges*** by *Alternating Series Test*.

***Exercise***

Determine if the alternating series converges or diverges 

***Solution***



Therefore; the given series ***diverges*** by n*th*-Term Test.

***Exercise***

Determine if the alternating series converges or diverges 

***Solution***





Therefore; the given series ***converges*** by *Alternating Series Test*.

***Exercise***

Determine if the alternating series converges or diverges 

***Solution***





Therefore; the given series ***converges*** by *Alternating Series Test*.

***Exercise***

Determine if the alternating series converges or diverges 

***Solution***



Therefore; the given series ***converges*** by n*th*-Term Test.

***Exercise***

Determine if the alternating series converges or diverges 

***Solution***







Therefore; the given series ***diverges*** by n*th*-Term Test.

***Exercise***

Determine if the alternating series converges or diverges 

***Solution***





Therefore; the given series ***converges*** by *Alternating Series Test*.

***Exercise***

Determine if the alternating series converges or diverges

***Solution***



Therefore; the given series ***diverges*** by n*th*-Term Test.

***Exercise***

Determine if the alternating series converges or diverges 

***Solution***







Therefore; the given series ***converges*** by *Alternating Series Test*.

***Exercise***

Determine if the alternating series converges or diverges 

***Solution***





Therefore; the given series ***converges*** by *Alternating Series Test*.

***Exercise***

Determine if the alternating series converges or diverges 

***Solution***





Therefore; the given series ***converges*** by *Alternating Series Test*.

***Exercise***

Determine if the alternating series converges or diverges 

***Solution***





Therefore; the given series ***converges*** by *Alternating Series Test*.

***Exercise***

Determine if the alternating series converges or diverges 

***Solution***





Therefore; the given series ***diverges*** by n*th*-Term Test.

***Exercise***

Determine if the series converge absolutely and if it converges or diverges 

***Solution***

 converges geometric since 

The given series ***converges*** absolutely.

***Exercise***

Determine if the series converge absolutely or conditionally, or diverges 

***Solution***





 converges geometric 

The given series ***converges*** absolutely by *Direct Comparison Test*.

***Exercise***

Determine if the series converge absolutely or conditionally, or diverges

***Solution***





The given series ***converges*** conditionally, but  is a divergent *p*-series.

***Exercise***

Determine if the series converge absolutely or conditionally, or diverges 

***Solution***

By Direct Comparison Test 

The given series ***converges*** *absolutely*.

***Exercise***

Determine if the series converge absolutely or conditionally, or diverges 

***Solution***



The given series ***diverges*** by the *nth*-Term Test.

***Exercise***

Determine if the series converge absolutely or conditionally, or diverges 

***Solution***









But by the Integral Test:







 ***diverges***.

***Exercise***

Determine if the series converge absolutely or conditionally, or diverges 

***Solution***



Which is a convergent *p*-series, since .

The given series ***converges*** ***absolutely*** by Direct Comparison Test.

***Exercise***

Determine if the series converge absolutely or conditionally, or diverges 

***Solution***

Let 







Since the harmonic series  ***diverges*** to infinity, then the given series doesn’t converge absolutely.

***Exercise***

Determine if the series converge absolutely or conditionally, or diverges 

***Solution***







Therefore; by the Ratio Test, the given series ***converges*** ***absolutely***.

***Exercise***

Determine if the series converge absolutely or conditionally, or diverges 

***Solution***

 converges by the alternating series test, since the terms alternate in sign (decrease in size) and approach 0.

 ***diverges*** to infinity, then the series converge conditionally only.

***Exercise***

Determine if the series converge absolutely or conditionally, or diverges 

***Solution***





 and  converges,

Therefore; the given series ***converges*** ***absolutely***.

***Exercise***

Determine if the series converge absolutely or conditionally, or diverges 

***Solution***





The given series ***diverges*** (since its terms do not approach 0.)

***Exercise***

Determine if the series converge absolutely or conditionally, or diverges 

***Solution***

 converges by alternating series test.

let , then







The series ***converges*** ***conditionally*** since  diverges to infinity by the *Integral Test*.

***Exercise***

Determine if the series converge absolutely or conditionally, or diverges

***Solution***





Therefore; the given series ***converges*** absolutely by the *Ratio Test*.

***Exercise***

Determine if the series converge absolutely or conditionally, or diverges 

***Solution***

, and since  is convergent geometric series, then the given series ***converges*** absolutely.

***Exercise***

Determine if the series converge absolutely or conditionally, or diverges 

***Solution***







The series converges by alternating series test but only conditionally.

The given series ***diverges*** to infinity.

***Exercise***

Determine if the series converge absolutely or conditionally, or diverges 

***Solution***







The given series ***diverges***.

***Exercise***

Determine if the series converge absolutely or conditionally, or diverges 

***Solution***





 for 

Since  diverges to infinity (it is a harmonic series), so does  by comparison.

The series converges conditionally by the *Alternating Series Test*.

***Exercise***

Determine if the series converge absolutely or conditionally, or diverges 

***Solution***

 converges Geometric series

The given series ***converges*** *absolutely*.

***Exercise***

Determine if the series converge absolutely or conditionally, or diverges 

***Solution***

 converges ***p-***series

The given series ***converges*** *absolutely*.

***Exercise***

Determine if the series converge absolutely or conditionally, or diverges 

***Solution***



 converges ***p-***series

The given series ***converges*** *absolutely*.

***Exercise***

Determine if the series converge absolutely or conditionally, or diverges 

***Solution***



 diverges by *Comparison*



 converges by *Alternating Series Test*

The given series ***converges*** *absolutely*.

***Exercise***

Determine if the series converge absolutely or conditionally, or diverges 

***Solution***



 converges by *Alternating Series Test*

 diverges by ***p-***series 

The given series ***converges*** *conditionally*.

***Exercise***

Determine if the series converge absolutely or conditionally, or diverges 

***Solution***

 converges by ***p-***series 

The given series ***converges*** *conditionally*.

***Exercise***

Determine if the series converge absolutely or conditionally, or diverges 

***Solution***



The given series ***diverges*** by the nth-Term Test.

***Exercise***

Determine if the series converge absolutely or conditionally, or diverges 

***Solution***



The given series ***diverges*** by the nth-Term Test.

***Exercise***

Determine if the series converge absolutely or conditionally, or diverges 

***Solution***





 By the Integral Test, the series diverges



 converges by *Alternating Series Test*

The given series ***converges*** *conditionally*.

***Exercise***

Determine if the series converge absolutely or conditionally, or diverges 

***Solution***

 Converges by geometric series 

 converges by Comparison Test

The given series ***converges*** *absolutely*.

***Exercise***

Determine if the series converge absolutely or conditionally, or diverges 

***Solution***



 converges by *p-*series 



 converges by Limit Comparison Test

The given series ***converges*** *absolutely*.

***Exercise***

Determine if the series converge absolutely or conditionally, or diverges 

***Solution***

 converges by *p-*series 



 converges by *Alternating Series Test*

The given series ***converges*** *absolutely*.

***Exercise***

Determine if the series converge absolutely or conditionally, or diverges 

***Solution***



 converges by *Alternating Series Test*

 diverges by ***p-***series 

 diverges by *Limit Comparison Test* using ***p-***series

The given series ***converges*** *conditionally*.

***Exercise***

Determine if the series converge absolutely or conditionally, or diverges 

***Solution***





 converges by Alternating Series Test

 diverges by a Limit Comparison to the divergent harmonic series





The given series ***converges*** *conditionally*.

***Exercise***

Determine if the series converge absolutely or conditionally, or diverges 

***Solution***



The given series ***diverges*** by the n*th*-Term Test.

***Exercise***

Determine if the series converge absolutely or conditionally, or diverges 

***Solution***





 converges by Alternating Series Test

 converges by *p-*series 

The given series ***converges*** *absolutely*.

***Exercise***

Determine if the series converge absolutely or conditionally, or diverges 

***Solution***





 converges by Alternating Series Test

 diverges by ***p-***series 

The given series ***converges*** *conditionally*.

***Exercise***

For what values of *x* does the series  converge absolutely? Converge conditionally?

***Solution***









 Then the series converges absolutely



 Then the series diverges (the term does approach zero)



If , the series  which converges conditionally (it is an alternating harmonic series).

If , the series  the series is harmonic which diverges.

Hence, the series ***converges*** ***absolutely*** on the open interval (3, 7), **converges conditionally** at , and ***diverges*** everywhere else.

***Exercise***

For what values of *x* does the series  converge absolutely? Converge conditionally? Diverge?

***Solution***

Using the ratio test







 Then the series converges absolutely



 Then the series diverges (the term does approach zero)



If , the series:







which converges absolutely (it is an alternating harmonic series).

If , the series







the series converges absolutely.

Hence, the series converges absolutely if and diverges everywhere else.

***Exercise***

For what values of *x* does the series  converge absolutely? Converge conditionally? Diverge?

***Solution***

Using the ratio test





|  |  |
| --- | --- |
|  |  |

|  |  |  |  |
| --- | --- | --- | --- |
| −∞ −2 −1 0 ∞ | | | |
| + | − | − | − |
| + | − | + | + |
|  |  | **−** | **−** |

If .

Hence  the series converges absolutely.

If , the series diverges.

If , the series is  which diverges

The series converges absolutely for , converges conditionally nowhere, and diverges for 

***Exercise***

For what values of *x* does the series  converge absolutely? Converge conditionally? Diverge?

***Solution***

Using the ratio test





If  Then the series converges absolutely

If  Then the series diverges

If 

If , the series  is harmonic which diverges.

If , the series  which converges absolutely (it is an alternating harmonic series).

Therefore; the series converges absolutely if and converges conditionally if  and diverges everywhere else.

***Exercise***

For what values of *x* does the series  converge absolutely? Converge conditionally? Diverge?

***Solution***

Using the ratio test





If  Then the series converges absolutely

If 

. Then the series diverges

If , the series 

 is harmonic which diverges.

If , the series  converges absolutely (it is an alternating harmonic series).

Therefore; the series converges absolutely if and , converges conditionally if  and diverges everywhere else.

***Exercise***

For what values of *x* does the series  converge absolutely? Converge conditionally? Diverge?

***Solution***

Using the ratio test





If , the given series converges absolutely.

If , the series



 converges absolutely (it is an alternating harmonic series).

If , the series



 is harmonic which diverges

Therefore; the series converges absolutely if and , converges conditionally if  and diverges everywhere else.

***Exercise***

For what values of *x* does the series  converge absolutely? Converge conditionally? Diverge?

***Solution***

Let by using the ratio test





If , the given series converges absolutely.

If , the series  converges absolutely.

If , the series  converges absolutely (*p-*series)

Therefore; the series ***converges absolutely*** if  and ***diverges*** everywhere else.

***Exercise***

For what values of *x* does the series  converge absolutely? Converge conditionally? Diverge?

***Solution***

Let by using the *Ratio Test*









If 



 the given series converges absolutely.

If 

The series  converges conditionally (Alternating test).

If 

The series  diverges.

Therefore; the series converges absolutely if ,  ***converges*** ***conditionally*** and ***diverges*** everywhere else.

***Exercise***

For what values of *x* does the series  converge absolutely? Converge conditionally? Diverge?

***Solution***

Let by using the ratio test





If 

, the given series converges absolutely.

If , the series

 converges conditionally (Alternating test).

Therefore; the series converges absolutely if ,  converges conditionally, diverges everywhere else, and undefined at .

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***









Therefore; the given series ***converges*** by the *Alternating Series Test*.

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***





Therefore; the given series ***converges*** by *Alternating Series Test*.

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***





Therefore; the given series ***converges*** by *Alternating Series Test*.

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***







Therefore; the given series ***converges*** ***absolutely*** by the *Ratio Test*.

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***







Therefore; the given series ***diverges*** by the *Ratio Test*.

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***





Therefore; the given series ***converges*** by *Alternating Series Test*.

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***









Therefore; the given series ***converges*** by *Alternating Series Test*.

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***

Which is *p-series* with 

Therefore; the given series ***converges*** by ***p****-series Test*.

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***







Therefore; the given series ***converges*** by the *Limit Comparison Test* with ***p****-series*.

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***





Therefore; the given series ***diverges*** by the Ratio Test.

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***



Therefore; the given series ***converges*** by the *Limit Comparison Test* with Geometric series 

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***

Therefore; the given series ***converges*** by *Geometric series* 

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***



Therefore; the given series ***diverges***.

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***



Therefore; the given series ***converges*** by *Geometric series* 

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***





Therefore; the given series ***converges*** conditionally by *Alternating Series Test*.

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***







Therefore; the given series ***converges*** absolutely by Alternating Series Test.

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***







Therefore; the given series ***diverges*** by *Integral Test*.

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***

Which is *p-series* with 

Therefore; the given series ***converges*** by ***p****-series Test*.

Let 









Therefore; the given series ***converges*** by *Integral Test*.

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***



It is *p-series* with  which diverges.

Therefore; the given series ***diverges*** by ***p****-series Test*.

Let 







Therefore; the given series ***diverges*** by *Integral Test*.

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***







Therefore; the given series ***diverges*** by *Divergence Test*.

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***



This is a Geometric series with , which converges.







Therefore; the given series ***converges*** by *Geometric series Test*.

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***









Therefore; the given series ***diverges*** by *Divergence Test*.









Because the limit is *ρ* = 1, we can’t decide from the *Ratio Test*

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***













Therefore; the given series ***converges*** by *Ratio Test*.

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***



Let 

diverges by ***p****−series* 







Therefore; the given series ***diverges*** by *Comparison Test*.

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***

Using Comparison test:







Let 



 converges by Geometric series with 

Therefore; the given series ***converges*** by *Comparison Test*.

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***





Therefore; the given series ***diverges*** by *Divergence Test*.

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***











Let 

converges by ***p****−series* 

Therefore; the given series ***converges*** by *Comparison Test*.

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***





Let 

diverges by ***p****−series* 

Therefore; the given series ***diverges*** by *Comparison Test*.

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***

Using Ratio Test:











Therefore; the given series ***converges*** by *Ratio Test*.

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***





converges by ***p****−series* 

Therefore; the given series ***converges*** by *Comparison Test*.

Let 

































Therefore; the given series ***converges*** by *Integral Test*.

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***









converges by ***p****−series* 

Therefore; the given series ***converges*** by *Comparison Test*.

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***

Using Ratio Test:













Therefore; the given series ***converges*** by *Ratio Test*.

Let 

|  |  |  |
| --- | --- | --- |
|  |  |  |
| **+** |  |  |
| **−** | 1 |  |







Therefore; the given series ***converges*** by *Integral Test*.

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***

Using Ratio Test:







Therefore; the given series ***converges*** by *Ratio Test*.

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***

Using Ratio Test:







Therefore; the given series ***converges*** by *Ratio Test*.

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***



Let  diverges by ***p****−series* 











Therefore; the given series ***diverges*** by *Limit Comparison Test*.

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***



Let 



 converges by Geometric series with 









Therefore; the given series ***converges*** by *Limit Comparison Test*.

Let 



















Therefore; the given series ***converges*** by *Integral Test*.

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***





Therefore; the given series ***diverges*** by *Divergence Test*.

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***



Let 



 converges by Geometric series with 







Therefore; the given series ***converges*** by *Limit Comparison Test*.

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***





Let  converges by ***p***−series 





Therefore; the given series ***converges absolutely*** by *Limit Comparison Test*.

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***



Therefore; the given series ***diverges*** by *Alternating series*.

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***

Using Ratio Test:







Therefore; the given series ***converges absolutely*** by *Ratio Test*.

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***









Therefore; the given series ***diverges*** by *Limit Comparison Test*.

But the series is decreasing, therefore; it is ***conditionally convergent***.

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***

Using Ratio Test:







Therefore; the given series ***converges absolutely*** by *Ratio Test*.

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***











Therefore; the given series ***diverges*** by the *Ratio Test*.

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***



Using Limit Comparison Test

let 



converges by Geometric series 







Therefore; the given series ***converges absolutely*** by the *Limit Comparison Test*.

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***



Let 









Therefore; the given series ***diverges absolutely*** by *Integral Test*.

However;





This series ***converges conditionally*** by the *Divergence Test*.

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***

Let 







As *x* gets larger, 

The given series decreases.









Therefore; the given series ***converges*** by *Alternating Series Test*.

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***



 ***√***



Therefore; the given series ***converges*** by *Alternating Series Test*.

Let 











Therefore; the given series ***converges absolutely*** by *Integral Test*.

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***



 ***√***



Therefore; the given series ***converges*** by *Alternating Series Test*.

However,





 ***diverges*** by ***p-****series* 

Therefore; the given series ***converges conditionally.***

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***



This is Geometric series with 







Therefore; the given series ***converges*** by *Geometric Series****.***

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***



Let 



 is Geometric series with 







Therefore; the given series ***converges*** by *Limit Comparison Test****.***

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***



Using the Ratio Test:







Therefore; the given series ***converges absolutely*** by *Ratio Test****.***

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***

Let 











Therefore; the given series ***converges*** by *Integral Test****.***

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***







Therefore; the given series ***diverges*** by *p-series* ***.***

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***



Therefore; the given series ***converges*** by *p-series* ***.***

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***





Therefore; the given series ***diverges*** by *Root Test****.***

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***





Therefore; the given series ***converges*** by *Root Test****.***

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***







 *diverges* by ***p****-series* with 







Therefore; the given series ***diverges*** by *Limit Comparison Test****.***

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***





 *converges* by ***p****-series* with 

Therefore; the given series ***converges*** by *Comparison Test****.***

Let 











Therefore; the given series ***converges*** by *Integral Test****.***

***Exercise***

Use a Riemann sum argument to show that 

Then for what values of *x* does the series  converge absolutely? Converge conditionally? Diverge? (*Use the ratio test first*)

***Solution***



= Sum of area of the shaded rectangles







Using the ratio test

Let 





If  ⇒ The given series converges absolutely.

If , then













Hence, the given series ***converges*** absolutely if  and ***diverges*** *elsewhere*.

***Exercise***

Let  be the *n*th partial sum of . Find the  and 

***Solution***

Since the series converges to 8, then 

Therefore; the partial sums converges to 8.



***Exercise***

It can be proved that if a series converges absolutely, then its terms may be summed in any order without changing the value of the series. However, if a series converges conditionally, then the value of the series depends on the order of summation. For example, the (conditionally convergent) alternating harmonic series has the value



Show that by rearranging the terms (so the sign pattern is ++−),



***Solution***













***Exercise***

A crew of workers is constructing a tunnel through a mountain. Understandably, the rate of construction decreases because rocks and earth must be removed a greater distance as the tunnel gets longer. Suppose that each week the crew digs 0.95 of the distance it dug the previous week. In the first week, the crew constructed 100 *m* of tunnel.

1. How far does the crew dig in 10 *weeks*? 20 *weeks*? *N* *weeks*?
2. What is the longest tunnel the crew can build at this rate?
3. The time required to dig 100 *m* increases by 10% each week, starting with 1 *week* to dig the first 100 *m*. Can the crew complete a 1.5 *km* tunnel in 10 *weeks*? Explain.

***Solution***

1. Let  be the amount of additional tunnel dug during week *n*. Then









So, the total distance dug in *N* weeks is







For 10 weeks: 



For 20 weeks: 



1. The longest possible tunnel is







1. The time required to dig  through 







The time required to dig 1500 *m* is:







So, it is not possible.

***Exercise***

Consider the alternating series



1. Write out the first ten terms of the series, group them in pairs, and show that the even partial sums of the series form the (divergent) harmonic series.
2. Show that 
3. Explain why the series diverges even though the terms of the series approach zero.

***Solution***

1. The first ten terms of the series are:



Suppose that 

Then the sum of the  term and the  term is



Then the sum of the even partial sums of the given series is 

1. 

Given  so that for  we have .

Also so that for , .

Let N be the larger of  or . Then for , we have  as desired.

1. The series can be seen to diverge because the even partial sums have limit . This does not contradict the alternating series test because the terms  are not nonincreasing.

***Exercise***

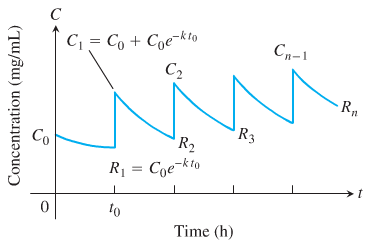
The concentration in the blood resulting from a single dose of a drug normally decreases with time as the drug is eliminated from the body. Doses may therefore need to be repeated periodically to keep the concentration from dropping below some particular level. One model for the effect of repeated doses gives the residual concentration just before the  does as



Where the change in concentration achievable by a single dose ,

 the elimination constant , and

 time between doses (*h*).



1. Write  in closed form as a single fraction, and find 
2. Calculate  and  for   and . How good as estimate of *R* is 
3. If  and , find the smallest *n* such that 

***Solution***

1. 











1. ***Given***:   and 















1. ***Given***:  and 















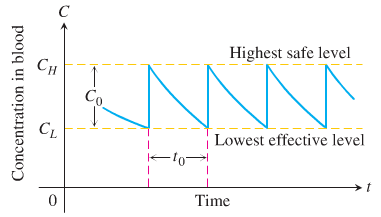






***Exercise***

If a drug is known to be ineffective below a concentration  and harmful above some higher concentration , one needs to find values of  and  that will produce a concentration that is safe (not above ) but effective (not below ).



We therefore want to find values for  and for which



Thus . The resulting equation simplifies to



To reach an effective level rapidly, one might administer a “loading” dose that would produce a concentration of . This could be followed every hours by a dose that raises the concentration by .

1. Verify the preceding equation for .
2. If  and the highest safe concentration is *e* times the lowest effective concentration, find the length of time between doses that will assure safe and effective concentrations.
3. Given  determine a scheme for administering the drug.
4. Suppose that  and the smallest effective concentration is 0.03 *mg/mL*. A single dose that produces a concentration of 0.1 *mg/mL* is administered. About how long will the drug remain effective?

***Solution***

1.  







1. 



1. Given 







A dose raises every 69.31 *hrs.* the concentration by 

1. 

